PRE-CALCULUS: by Finney,Demana,Watts and Kennedy

Chapter 2: Polynomial, Power, and Rational Functions

2.5: Complex Zeros and the Fundamental Theorem of Algebra

What you'll Learn About

Write the polynomial in standard form, and identify the zeros of the function and the x-intercepts.

a) $(x - 4i)(x + 4i)$

$x^2 + 4ix - 4ix - 4i^2$

$x^2 - 16(-1)$

$x^2 + 16$

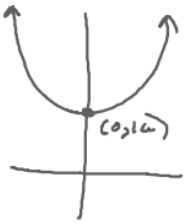
Zeros: $x - 4i = 0$, $x = 4i$ ←

$x + 4i = 0$, $x = -4i$ ←

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

x-intercepts: None



b) $(x - 3)(x - \sqrt{4}i)(x + \sqrt{4}i)$

$\sqrt{4}i = 2i$

$(x-3)(x-2i)(x+2i)$

$(x-3)(x^2 + 2ix - 2ix - 4i^2)$

$(x-3)(x^2+4)$

$x^3 - 3x^2 + 4x - 12$

Zeros: $x - 3 = 0$, $x = 3$; $x - 2i = 0$, $x = 2i$; $x + 2i = 0$, $x = -2i$

x-intercept: (3,0)

c) $x(x - 3)(x - 2 - i)(x - 2 + i)$

$((x-2) - i)((x-2)+i)$

$x^2 - 2x + ix$
$-2x + 4 - ix + 2i - i^2$

$x^2 - 4x + 4 - i^2$

$x(x-3)(x^2 - 4x + 5)$

$x(x^3 - 7x^2 + 17x - 15)$

$x^4 - 7x^3 + 17x^2 - 15x$

Zeros: $x = 0$; $x - 3 = 0$, $x = 3$; $x - 2 - i = 0$, $x = 2 + i$; $x - 2 + i = 0$, $x = 2 - i$

x-intercepts: (0,0) (3,0)

$x^3 - 4x^2 + 5x$
$-3x^2 + 12x - 15$

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed.

a) 2, 5i, and -6i

$2, 5i, -5i, -6i, 6i$

$(x-2)(x-5i)(x+5i)(x+6i)(x-6i)$

Real Factored Form →

$(x-2)(x^2+25)(x^2+36)$

$(x-2)(x^4+61x^2+900)$

$(x-5i)(x+5i)$

$x^2+5ix-5ix-25i^2$

$x^2-25(-1)$

x^2+25

$x^5 + 61x^3 + 900x$

$-2x^4 - 122x^2 - 1800$

$x^5-2x^4+61x^3-122x^2+900x-1800$

b) -2, 3, and 2 – i

$x=-2, 3, 2-i, 2+i$

$(x+2)(x-3)(x-(2-i))(x-(2+i))$

$(x-2+i)(x-2-i)$

$(x+2)(x-3)(x^2-4x+5)$

$x^2-2x-xi$

$-2x + 4 + 2i$

$xi - 2i - i^2$

$x^2-4x+4-i^2$

c) -4, 2 + 3i

$(x+4)(x-(2+3i))(x-(2-3i))$

$(x-2-3i)(x-2+3i)$

$(x+4)(x^2-4x+13)$

$x^2-2x+3ix$

$-2x + 4 - 6i$

$-3ix + 6i - 9i^2$

$x^2-4x+4-9i^2$

$x^2-4x+13$

$(x-3)(x-3)$

x^2-6x+9

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include those listed. Then sketch a graph and discuss what you notice.

a) 3 (multiplicity 2), - 4 (multiplicity 3)

$x=3 \qquad x=-4$

$(x-3)^2\,(x+4)^3$ ←

$(x^2-6x+9)(x^3+12x^2+48x+64)$

$(x+4)^3 \neq x^3+64$

$(x+4)(x+4)(x+4)$

$(x+4)(x^2+8x+16)$

x^3+8x^2+16x

$4x^2+32x+64$

b) 3 (multiplicity 3), - 4 (multiplicity 1)

$(x-3)^3(x+4)$

c) 5 (multiplicity 2), $2+i$ (multiplicity 1) $2-i$ (mult of 1)

$(x-5)^2(x-(2-i))(x-(2+i))$

2.5

5-15 odd

5, 11, 15 Standard

7, 9, 13 — Factored